ARTIFICIAL INTELLIGENCE & MACHINE LEARNING

UNIT II

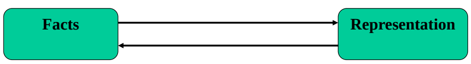
Learning Objectives

Knowledge Representation and Reasoning:

1. Approaches to knowledge representation
2. Propositional Logic
3. First Order Predicate Logic
4. Inference Rules
5. (Modus Ponen, Modus Tollen, Resolution, And elimination, Syllogism
6. Production Rules
7. Types of knowledge
8. Reasoning: Forward and backward reasoning
9. Non-monotonic Reasoning
10. Reasoning with uncertainties

Approaches to knowledge representation

- A variety of ways representing knowledge have been used in AI programs.
  - A knowledge level: where facts has been described.
  - The symbol level: Where representation of the objects at knowledge level are defined in terms of symbols that can be manipulated by the program to achieve the goal.



- The Knowledge representation models/mechanisms are often based on:
  - Logic
  - Rules
  - Frames
  - Semantic Network

Approaches to knowledge representation (contd.)

- The base for Knowledge based agent is termed as Knowledge Base (KB)
- This KB consists of set of sentences that represents as knowledge representation language.
- When the sentence is taken without being derived from other sentences, we call it as **AXIOMS**.
- There must be a way to add more sentences to knowledge base, and to query what is already known.
  - **TELL and ASK**

Approaches to knowledge representation (contd.)

- Inference: Deriving new sentences from old one.
- Inference must obey the requirements that when one **ASK** a question of the knowledge base, the answer should follow from what has been **TOLD** to the KB.
- A knowledge based agent can be built by TELLING ut what it needs to know.
- Starting with empty KB, the agent can tell sentence one by one until the agent know how to operate its environment. → **Declarative approach.**
- When encoding is done for desired behavior properly and directly→ **Procedural approach.**

Logic

- Syntax: The structure of sentence
- Semantics: the way in which truth of sentence is determined.

$X+Y=4$

- The **atomic sentences** consist of a single proposition symbol. P,Q,$W_{1,3}$ and *FacingEast.*
- Complex sentence are constructed from simpler sentences, using parentheses and operators → **Logical connectives.**

Logic (contd.)

- There are 5 common connectives:

$\neg$ (not). A sentence such as $\neg W_{1,3}$ is called the **negation** of $W_{1,3}$. A **literal** is either an atomic sentence (a **positive literal**) or a negated atomic sentence (a **negative literal**).

$\wedge$ (and). A sentence whose main connective is $\wedge$, such as $W_{1,3} \wedge P_{3,1}$, is called a **conjunction**; its parts are the **conjuncts**. (The $\wedge$ looks like an "A" for "And.")

$\vee$ (or). A sentence whose main connective is $\vee$, such as $(W_{1,3} \wedge P_{3,1}) \vee W_{2,2}$, is a **disjunction**; its parts are **disjuncts**—in this example, $(W_{1,3} \wedge P_{3,1})$ and $W_{2,2}$.

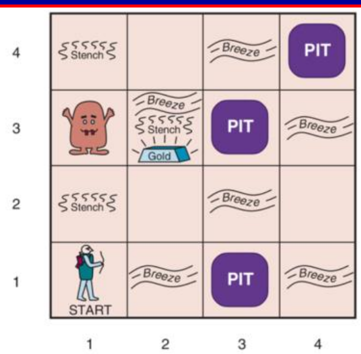
$\Rightarrow$ (implies). A sentence such as $(W_{1,3} \wedge P_{3,1}) \Rightarrow \neg W_{2,2}$ is called an **implication** (or conditional). Its **premise** or **antecedent** is $(W_{1,3} \wedge P_{3,1})$, and its **conclusion** or **consequent** is $\neg W_{2,2}$. Implications are also known as **rules** or **if–then** statements. The implication symbol is sometimes written in other books as $\supset$ or $\rightarrow$.

$\Leftrightarrow$ (if and only if). The sentence $W_{1,3} \Leftrightarrow \neg W_{2,2}$ is a **biconditional**.

Logic (contd.)

- For atomic sentences:
  - True is true in every model and false is false in every model.
- For complex sentences:
  - $\neg P$ is true iff P is false in m.
  - $P \wedge Q$ is true iff both P and Q are true in m.
  - $P \vee Q$ is true iff either P or Q is true in m.
  - $P \Rightarrow Q$ is true unless P is true and Q is false in m.
  - $P \Leftrightarrow Q$ is true iff P and Q are both true or both false in m.

A Typical Wumpus World!!!



Learning Objectives

(a) (b)

A simple Knowledge Base

$P_{x,y}$ is true if there is a pit in $[x,y]$.

$W_{x,y}$ is true if there is a wumpus in $[x,y]$, dead or alive.

$B_{x,y}$ is true if there is a breeze in $[x,y]$.

$S_{x,y}$ is true if there is a stench in $[x,y]$.

$L_{x,y}$ is true if the agent is in location $[x,y]$.

A simple Knowledge Base (contd.)

- There is no pit in [1,1]

$$R_1 : \quad \neg P_{1,1}$$

- A square is breezy if and only if there is a pit in a neighboring square. Now this has to be stated for every square.

$$R_2 : \quad B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1}) .$$

Write a relation if square $B_{2,1}$ is breezy

Standard Logical Equivalences

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg \alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \beta \Rightarrow \neg \alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg \alpha \vee \neg \beta) && \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg \alpha \wedge \neg \beta) && \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Inference and proofs

- The rules can be applied to derive a proof→ a chain of conclusion that leads to the desired goal.
- The best known rule is **MODUS PONENS.**

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

- Whenever any sentence of the form $\alpha \Rightarrow \beta$ and α are given, then the sentence β can be inferred.
- **And-Elimination** $\dfrac{\alpha \wedge \beta}{\alpha}$

First Order Predicate Logic

- In propositional logic, we can only represent the facts, which are **either true or false**.
- The propositional logic has very limited expressive power.
- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- First-order logic is also known as **Predicate logic or First-order predicate logic.** First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.

Using First Order Predicate Logic

- In knowledge representation, a domain is just some part of the world about which we wish to express some knowledge.
- Sentences are added to KB using **TELL**, just like we did in propositional logic. These are termed as **assertions.**

$$\text{TELL}(KB, King(John)) .$$
$$\text{TELL}(KB, Person(Richard)) .$$
$$\text{TELL}(KB, \forall x\ King(x) \Rightarrow Person(x)) .$$

- Ask question of the KB using **ASK.** These are termed as queries or goals.

ASK(KB,King(John))

Quantifier

- Universal Quantifier ($\forall$)

$$\forall x\ \ King(x) \Rightarrow Person(x).$$

  - The sentence says, "For all x, if x is a king, then x is a person." The symbol x is called a variable.
- Existential quantification ($\exists$)
  - we can make a statement about some object without naming it, by using an existential quantifier

Statements

1. All boys like football
2. Some girls like basketball
3. Some girls hate pumpkin
4. All boys hate potato
5. Every person who buys a PlayStation is smart
6. No person buys expensive gifts
7. Not all students like both mathematics and Science

Questions

1. **Not all students like both Mathematics and Science.**
2. **Only one student failed in Mathematics.**
3. **None of my friends are perfect.**
4. John does not love anyone
5. Everyone loves someone.

Questions

"None of my friends are perfect."

(A) $\exists x(F(x) \wedge \neg P(x))$

(B) $\exists x(\neg F(x) \wedge P(x))$

(C) $\exists x(\neg F(x) \wedge \neg P(x))$

(D) $\neg \exists x(F(x) \wedge P(x))$

Clause

- Definite Clause: A clause which is a disjunction of literals with **exactly one positive literal** is known as a definite clause or strict horn clause.
- Horn Clause: A clause which is a disjunction of literals with **at most one positive literal** is known as horn clause.

Hence all the definite clauses are horn clauses

Forward Reasoning

- **Forward reasoning** is a process in artificial intelligence that finds all the possible solutions of a problem based on the initial data and facts.
- The forward reasoning is a **data-driven task** as it begins with new data.
- In forward reasoning, the first step is that the system is given one or more constraints.
- Forward reasoning follows the **Bottom-up** approach.

Forward Reasoning (Contd.)

- Rules

A & C →E

A & E →G

B →E

G →D

Prove A & B →D

DB== AB present

Backward Reasoning

- It is a **goal- driven** task.
- Backward reasoning begins with some goal
- Backward reasoning is a top-down approach.
- Rules:

F & B→Z

C & D →F

A →D

In facts : A E B C

Reasoning under Uncertainties

- Till now, we have learned knowledge representation using first-order logic and propositional logic with certainty, which means **we were sure about the predicates.**
- With this knowledge representation, we might write A→B, which means if A is true then B is true, but consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation is called uncertainty.
- So to represent uncertain knowledge, where we are not sure about the predicates, we need **uncertain reasoning or probabilistic reasoning.**

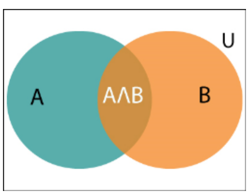
Probabilistic Reasoning

- **A knowledge representation where the concept of probability is applied to indicate uncertainty.**
- **Need of probabilistic reasoning in AI:**
  - **When there are unpredictable outcomes.**
  - **When specifications or possibilities of predicates becomes too large to handle.**
  - **When an unknown error occurs during an experiment.**

Probability: Probability can be defined as a chance that an uncertain event will occur. It is the numerical measure of the likelihood that an event will occur. The value of probability always remains between 0 and 1 that represent ideal uncertainties.

Conditional Probability

Probabilistic Reasoning



I roll a fair die. Let A be the event that the outcome is an odd number, i.e., $A = \{1,3,5\}$. Also let B be the event that the outcome is less than or equal to 3, i.e., $B = \{1,2,3\}$. What is the probability of A, $P(A)$? What is the probability of A given B, $P(A|B)$?

Bayes’ theorem in Artificial Intelligence

- Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.
- It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

Bayes’ theorem in Artificial Intelligence

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

- P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.
- P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.
- P(A) is called the **prior probability**, probability of hypothesis before considering the evidence
- P(B) is called **marginal probability**, pure probability of an evidence.

Bayes’ theorem in Artificial Intelligence

Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is 4/52, then calculate posterior probability P(King|Face), which means the drawn face card is a king card.

Reasoning

Differentiate between Monotonic and Non-Monotonic Reasoning

