2017 Question Paper

END TERM EXAMINATION

FIRST SEMESTER [MCA] NOVEMBER-DECEMBER 2017

Paper Code: MCA-105	*Subject: Discrete Mathematics*
Time: 3 Hours	*Maximum Marks: 75*

Note: ***Attempt any five questions including Q.no.1 which is compulsory. Select one question from each Unit.***

Q1 (a) Prove that $(A - C) \cap (C - B) = \Phi$ analytically, where A, B, C are sets.

(b) Let Z^+ be set of +ve integers. Let R be a relation defined on Z^+ as follows aRb $\Leftrightarrow$ a divides b. Give the type of relation R.

(c) What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$.

(d) Prove that $p \rightarrow q = \sim p \vee q$.

(e) Let Z^+ be the set of +ve integers, show that $(Z^+, \leq)$ is a Poset.

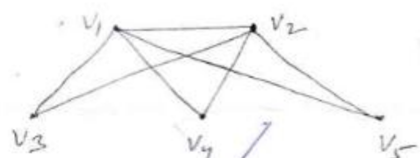
(f) Let $B(+, ., ', 0, 1)$ be a Boolean algebra. Show that for any $p, q \epsilon B, p'q + p + p'q' = 1$

(g) What are applications of number theory in computer science?

(h) Let (G, .) be a group. Prove that $(xy)^{-1} = y^{-1}x^{-1}$.

(i) Define Hamiltonian circuit with example.

(j) Find the adjacency matrix of the graph shown below. **(2.5x10=25)**



Unit-I

Q2 (a) In how many ways can a party of seven persons arrange themselves around a circular table? Also find number of ways in which they can arrange themselves in a queue. **(3.5)**

(b) Find the minimum number of students in a class so that three of them are born in the same month. **(3.5)**

(c) Define transitive closure of a relation R on set A. Find the transitive closure of the relation R given by $R = \{(1,1), (1,3), (3,1), (3,2), (2,2)\}$ defined on a set A = $\{1, 2, 3\}$. **(5.5)**

Q3 (a) Let A be set of +ve integer. Let R be a relation on A defined as $(a, b) \epsilon R \Leftrightarrow (a - b)$ is divisible by $m \neq 0$, where m is +ve integer. Show that R is an equivalence relation. **(3.5)**

(b) Show by mathematical induction:

$$\forall\, n \in N \left[\sum_{i=0}^{n} i = \frac{n(n+1)}{2}\right].$$ **(3.5)**

(c) Find the conclusion of the following hypothesis: **(5.5)**

(i) It is not sunny this afternoon and it is colder than yesterday.
(ii) We will go swimming only if it is sunny.
(iii) If we do not go swimming, then we will take a canoe trip.
(iv) If we take a canoe trip, then we will be home by sunset.

P.T.O.

Unit-II

Q4 (a) Discuss Hasse diagram to represent a poset. What are LUB and GLB. Illustrate through an example. **(3.5)**

(b) Define a distributive lattice. Consider the lattice a<b, a<c, b<d, d<e, c<e. Show that this lattice is not distributive. **(3.5)**

(c) Let (L, ≤) be a bounded distributed lattice with 1 and 0 as unit and zero elements of L respectively. **(5.5)**

(i) Prove the DeMorgan's Law.
(ii) Show that if the complement of an element in L exists then it is unique.

Q5 (a) Let D_{20} be the set of all divisors of 20. Prove it is a lattice but not finite Boolean algebra. **(3.5)**

(b) Simplify the Boolean expression $E = xyz + xyz' + xy'z + x'yz' + x'y'z$. **(3.5)**

(c) Find the solution of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$. Find n ≥ 2. **(5.5)**

Unit-III

Q6 (a) What is a public key cryptography? Explain RSA Cryptosystem in detail. **(6.25)**

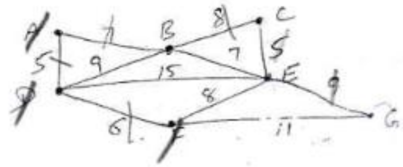
(b) Explain Euclidean algorithm to find the gcd of two numbers by taking example. **(6.25)**

Q7 (a) State and prove Lagrange's theorem for groups. **(6.25)**

(b) Let (G, .) be a group. Prove that $(a.b)^{-1} = b^{-1}\,a^{-1}$ for $a, b \in G$. Also show that $((a.b)^{-1})^{-1} = a.b$. **(6.25)**

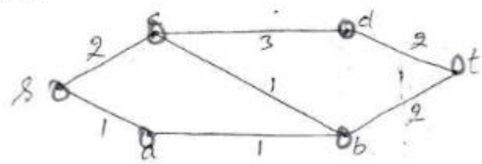
Unit-IV

Q8 (a) Using Prim's algorithm, find minimal spanning tree from the following graph. **(8.5)**



(b) Show that the number of vertices with odd degree in a graph is always even. **(4)**

Q9 (a) Write the steps for finding the shortest path between two vertices of a graph using Dijsktra's method. Hence find the shortest path between node s and t. **(8.5)**



s a b c d =

(b) State and prove Eulers theorem for planner graph. **(4)**

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